UNIT 4: PROBABILITY!

Ch. 14 -- 17

HAVE OUT YOUR HW WORKSHEET

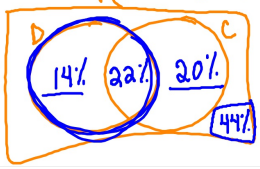
ANSWERS:

1) 4/52 = 1/13
2) 26/52 = 1/2
3) 2/52
4) 2/52
5) 1/52
6) 8/52
7) 28/52
8) 6/52

9) 3/15 = 1/5 $(4/15)(3/14)$
10) 8/15 $(2/15)(3/14)$
11) 2/35
12) 1/35
13) 9/15 = 3/5
14) 0

15) 56% $= P(D \cup C)$
16) 44%
17) 61.11% $P(C|D) = \frac{22\%}{36\%}$

$P(D) = 36\%$
$P(C) = 42\%$
$P(D \cap C) = 22\%$



Experimental Probability-

The actual probability of an outcome in an experiment (what did happen)

Ex: If I toss a coin 30 times, and get 12 heads, what's the **experimental** prob. of getting heads? $P(H) = 12/30 = 0.4 = 40\%$

$\hat{P}$

Theoretical Probability-

The expected probability of an outcome (what should happen)

P

Ex: Using the same coin tossing situation above, what's the **theoretical** prob. of getting heads? $P(H) = 50\% = 15/30 = 0.5$

The Law of Large Numbers:

As the number of trials increases, the experimental probability gets (should) closer to the theoretical probability.

Ex: Coin flips

$n = 30$ $P(H) = 40\%$

$n = 100$ $P(H) = 45\%$

$n = 10{,}000$ $P(H) = 50\%$

Sample Space-

List of all possible outcomes

Example: What is the sample space when rolling 2 dice?

#2-12

(1,1) (2,1) ..... (6,1)
(1,2) (2,2)
(1,3)
(1,6) (6,6)

Probability Notation:

- A, B, C, etc. = events P(Heads)
- P(A) = probability of A happening
- S = sample space $P(S_p)$

Probability Rules: BASIC

* Let A^c = the complement of event A (event A ***NOT*** happening)

The 3 Probability Rules

(1) $0 \le P(A) \le 1$

(2) P(S) = 1 = 100%

(3) $P(A^c) = 1$ -- P(A)

$P(6) = 1/6$

$P(6^c) = 5/6$

Example 1: If the probability of hitting a homerun is 30%, what's the probability of not hitting a homerun?

$P(H) = 30\%$ $P(H^c) = 70\%$

Example 2: If there are only 8 different blood types, fill in the chart below:

Type:	A+	A-	B+	B-	AB+	AB-	O+	O-
	0.16	0.14	0.19	0.17	?0.06	0.07	0.10	0.11

Example 3: Las Vegas Zeke, when asked to predict the ACC basketball Champion, follows the modern practice of giving probabilistic predictions. He says, "UNC's probability of winning is twice Duke's. NC State and UVA each have probability 0.1 of winning, but Duke's probability is three times that. Nobody else has a chance." Has Zeke given a legitimate assignment of probabilities to all the teams in the conference? Why or why not?

$P(UNC) = 0.6$
$P(Duke) = 0.3$
$P(NCSt.) = 0.1$
$P(UVA) = 0.1$
$P(VT) = 0/1.1$

SYMBOLS:

UNION

Meaning: addition, OR Symbol: $\cup$

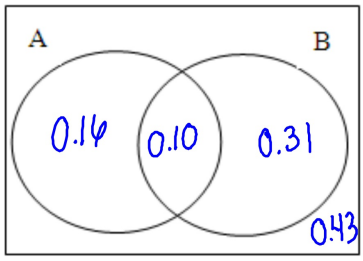
INTERSECTION

Meaning: overlap, AND Symbol: $\cap$

VENN DIAGRAMS

- Use shapes to represent events
- Box around the shapes to represent the sample space

Example: P(A)= 0.26 and P(B)= 0.41 and P(A ∩ B)= 0.10



Example: P(A)= 0.32 and P(B)= 0.61 and P(A n B)= 0.22

$P(A \cap B)$

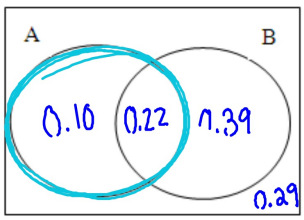
P(A U B)= $P(A \text{ or } B) = 0.71$

$P(A^c \cup B) = P(A^c \text{ or } B) = 0.90$

$P(A^c \cup B^c) = 0.78$ OR

$P(A^c \cap B) = 0.39$

$P(A^c \cap B^c) = 0.29$ AND



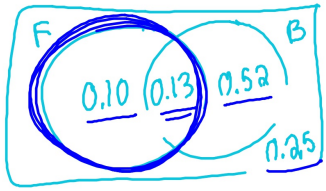
$P(B|A) = \frac{0.22}{0.32} = \frac{2}{52}$

Example: Real estate ads suggest that 23% of homes for sale have fireplaces, 65% have a second bathroom, and 13% have both features. What is the probability that a home for sale has …

- A second bathroom and a fireplace? $P(B \cap F) = 0.13$
- Neither a second bathroom nor a fireplace? $P(B^c \cap F^c) = 0.25$
- A second bathroom but no fireplace? $P(B \cap F^c) = 0.52$

$P(F) = 23\%$ $P(B) = 65\%$ $P(F \cap B) = 13\%$

$P(B|F) = \frac{0.13}{0.23} = 0.56$



PROBABILITY RULES: UNIONS (OR)

Example: For a deck of cards..... When picking 1 card, what is the probability of picking a red card OR a face card?

$P(R \cup F) = \frac{26}{52} + \frac{12}{52} = \frac{38}{52}$

General Rule:

* $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

*on formula sheet

Special Case:

What if A and B don't overlap? So, $P(A \cap B) = 0$

Example: Picking a red card and a spade $P(R \cup Sp)$

This is called **Disjoint = mutually exclusive =**

Two events are disjoint if they have no outcomes in common

Disjoint events: $P(A+ \cup A- \cup AB+ \cup AB-)$

$P(A \cup B) = P(A) + P(B)$

ex: blood types

$P(A \cup B \cup C \cup \ldots) = P(A) + P(B) + P(C) + \ldots$

Example: We are picking one card out of a standard 52-card deck. What's the probability of picking...

A heart or a spade? $P(H \cup Sp) = P(H) + P(Sp) = \frac{13}{52} + \frac{13}{52} = \frac{26}{52}$

A 3 or a spade? $P(3 \cup Sp) = P(3) + P(Sp) - P(3 \cap Sp) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52}$

A Queen or a King?

A red card or a heart? $P(R \cup H) = P(R) + P(H) - P(R \cap H) = \frac{26}{52} + \frac{13}{52} - \frac{13}{52} = \frac{26}{52}$

A black card or a face card?

subset

Example: The probability of event G is 0.25 and event K is 0.34. $P(G \cap K) = 0.1$. What is $P(G \cup K)$?

$P(G) = 0.25$

$P(K) = 0.34$

$P(G \cup K) = P(G) + P(K) - P(G \cap K)$

$= 0.49$

PROBABILITY RULES: INTERSECTIONS & CONDITIONALS

CONDITIONAL:

P(B|A) = Prob of B happening given that A happened already

$P(B|A) = \frac{P(B \cap A)}{P(A)}$. $P(A)$ $P(B|A) \neq P(A|B)$

A = 1st event that happened ***B = 2nd event that happened***

REARRANGE --> INTERSECTION:

$P(A \cap B) =$ $P(B|A) \cdot P(A)$

INDEPENDENCE = when one event happening does not affect the chance of a second event happening

Examples: picking things WITH replacement, coin flips, die rolls

If $P(B|A) = P(B)$ then A and B are independent.

$P(2|6) = P(2)$ $P(3|H) = P(3)$

Independent events:

$P(A \cap B) =$ $P(B|A) \cdot P(A)$ $P(B) \cdot P(A)$

$P(A \cap B \cap C \ldots) = P(A) \cdot P(B) \cdot P(C)$

Example: P(J) = 0.23 and P(B) = 0.67 and P(J | B) = 0.15.

(a) What is P(J n B)?

$P(B|A)=\frac{P(A\cap B)}{P(A)}$ $P(J|B)=\frac{P(J\cap B)}{P(B)}$

$0.15=\frac{P(J\cap B)}{0.67}$

$P(J\cap B)=0.1005$

(b) Are J and B independent?

$P(B|A)=P(B)$

$P(J|B)=P(J)$ → $0.15\neq 0.23$ NO

(c) Are J and B disjoint?

$P(J\cap B)\neq 0$

$0.1005\neq 0$

NO

Example: P(A) = 0.45 and P(C) = 0.39 and P(A n C) = 0.22.

(a) What is P(A | C)? $=\frac{P(A\cap C)}{P(C)}=\frac{0.22}{0.39}=0.5641$

(b) What is P(C | A)? $=\frac{P(C\cap A)}{P(A)}=\frac{0.22}{0.45}=0.4889$

(c) Are C and A independent?

$P(A|C)=P(A)$?

$0.5641\neq 0.45$ NO

(d) Are C and A disjoint?

$P(A\cap C)=0$?

$0.22\neq 0$ NO

Example: Look at the following table about grade level and favorite type of pet and answer the probability questions:

	Frosh	Soph	Junior	Senior	
Dog	14	18	22	16	70
Cat	8	11	13	15	47
Other	12	14	10	9	45
	34	43	45	40	***162***

(a) If someone is a sophomore, what is the prob they like Dogs?

(b) Given that someone likes Cats, what is the prob they are a junior?

(c) We pick a frosh at random. What is the prob that they like Other?

Example: Look at the following table about grade level and favorite type of pet and answer the probability questions:

	Frosh	Soph	Junior	Senior	
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Other	12	14	10	9	45
	34	43	45	40	***162***

(a) If someone is a sophomore, what is the prob they like Dogs?

P(D|So) = 18/43

(b) Given that someone likes Cats, what is the prob they are a junior?

P(Jr|C) = 13/47

(c) We pick a frosh at random. What is the prob that they like Other?

P(O|Fr) = 12/34

Example: In a bag of gummy bears, 12 were red, 8 were green, 9 were orange, and 10 were yellow. Find the probability that you …

- Eat a red bear then a green bear
- Eat a red bear and a green bear
- Eat 3 orange bears

Example: Suppose 40% of cars in your area are manufactured in the US, 30% in Japan, 10% in Germany, and 20% in other countries. If cars are selected at random, find the probability that …

- A car is not US made
- It is made in Japan or Germany
- You see two in a row from Japan
- None of the three cars came from Germany
- At least one of three cars is US made
- The first Japanese car is the fourth one you choose.

Example: Suppose 40% of cars in your area are manufactured in the US, 30% in Japan, 10% in Germany, and 20% in other countries. If cars are selected at random, find the probability that …

P(US) = 0.40
P(J) = 0.30
P(G) = 0.10
P(O) = 0.20

- A car is not US made $P(US^c) = 0.60$
- It is made in Japan or Germany

$P(J \cup G) = 0.30 + 0.10 = 0.40$

- You see two in a row from Japan

$P(J \cap J) = (0.30)(0.30) = 0.09$

- None of the three cars came from Germany

$P(G^c \cap G^c \cap G^c) = (0.90)(0.90)(0.90) = 0.729$

- At least one of three cars is US made

$1 - P(US^c \cap US^c \cap US^c) = 1-(0.60*0.60*0.60) = 0.784$

- The first Japanese car is the fourth one you choose.

$P(J^c \cap J^c \cap J^c \cap J) = (0.70*0.70*0.70*0.30) = 0.1029$

Example: At a local track meet 62% of the participants were girls, 29% of the participants were from private schools, and 18% of the participants were girls from private schools. What's the prob…

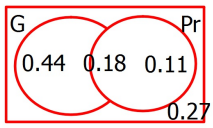
- That a student was a girl or from a private school?
- That a girl was from a private school?
- That a randomly selected public school student was a girl?
- Is being a girl and being from a private school mutually exclusive?
- Is being a girl and being from a private school independent?

Example: At a local track meet 62% of the participants were girls, 29% of the participants were from private schools, and 18% of the participants were girls from private schools. What's the prob…

$P(G) = 0.62$ $P(Pr) = 0.29$ $P(G \cap PR) = 0.18$

- That a student was a girl or from a private school?

$P(G \cup Pr) = 0.73$



- That a girl was from a private school?

$P(Pr|G) = 0.18/0.62 = 0.2903$

- That a randomly selected public school student was a girl?

$P(G|Pr^c) = 0.44/0.71 = 0.6197$

- Is being a girl and being from a private school mutually exclusive?

NO. $P(G \cap Pr) \neq 0$

- Is being a girl and being from a private school independent?

NO. $P(Pr|G) \neq P(Pr)$ or Yes, $P(Pr|G)$ approx $= P(Pr)$

Probability Rules

$0 \leq P(A) \leq 1$

$P(S) = 1$

$P(A^c) = 1 - P(A)$

UNIONS: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A and B are **disjoint**, **$P(A \cup B) = P(A) + P(B)$** b/c $P(A \cap B) = 0$

INTERSECTIONS: $P(A \cap B) = P(A) * P(B|A)$

If A and B are **independent**, **$P(A \cap B) = P(A) * P(B)$** b/c $P(B|A) = P(B)$

CONDITIONAL: $P(B|A) = \frac{P(A \cap B)}{P(A)}$ $P(A) > 0$